Capital adequacy for credit risk: A practical exercise
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Introduction
The financial crisis and its effect on the real economy initiated a debate at the beginning of 2009 on the need to change financial industry regulation in order to prevent new crises, mitigate systemic risk and develop a balanced framework for competition.

This process is resulting in a number of consultative documents and regulatory proposals by the Basel Committee, FSB, FSF, IASC, CEBS, and other international agencies and forums which, once calibrated on the basis of impact analysis studies (QIS), will gradually enter into force until their full implementation in 2012.

In particular, the Basel Committee is developing different proposals in relation to the three Basel II pillars. Within the framework of Pillar 2, these proposals emphasize the relevance of the capital measurement and planning process in assessing capital adequacy, as a fundamental part of the risk management and control function in a financial institution.

This process requires entities to engage in a capital self-assessment exercise whereby, based on the entity’s risk profile and on the current economic and financial environment, all material risks affecting the institution are identified and assessed in an integrated manner in order to reach a conclusion on its capital adequacy status. This process also involves performing a number of rigorous stress testing exercises prospectively, with a view to detecting possible developments or changes in market conditions that could negatively affect the entity.

Prior to the financial crisis, the scenarios used by entities in their capital planning processes tended to be continuous in nature, therefore stress tests did not always reflect a possible economic downturn and its impact on the solvency of institutions.

Institutions that are more advanced in risk management terms have developed internal risk measurement and management models and conducted planning exercises over the longer term, using three-year projections of the core capital base and capital consumption levels. This has made it possible to produce more accurate estimates of future solvency and to define contingency plans.

Within the context described above, this study provides an analysis of credit risk capital requirements under different scenarios and risk parameter assumptions in order to assess how these scenarios affect the regulatory capital model as well as economic capital consumption.

Risk parameters comprise the various quantitative aspects to which capital is sensitive, such as through-the-cycle adjusted PD, procyclicality, stochastic LGD, rating migration or concentration.

Such simulation exercise supplements the stress tests commonly undertaken by entities as part of their capital measurement and planning process, whereby, given a number of macroeconomic scenarios, specific risk parameters are set for each scenario and capital requirements are estimated.

For the purposes of the study, regulatory capital requirements were estimated, as was the figure that would result from using an economic capital model based on methodology commonly applied by IRB entities. Such estimates were made on the basis of standard portfolios from the Mortgage Loan (Retail) and SME segments of up to EUR 100 million turnover, since these portfolios are more representative of Spanish financial institutions in terms of exposure.

This document is structured into the following sections:

- Executive summary of the conclusions reached in the study.
- Description of the portfolio characteristics used in the analyses.
- Methodological foundations used.
- Tests performed and analysis of findings.

1Financial Stability Board.
2Financial Stability Forum.
3International Accounting Standards Committee.
4Committee of European Banking Supervisors.
5Quantitative Impact Survey.
6“Consultative proposals to strengthen the resilience of the banking sector”, by the Basel Committee of December 17, 2009.
7Pillar 1 (Minimum capital requirements), Pillar 2 (Self-assessment of capital adequacy and supervisory review) and Pillar 3 (Market discipline).
8Entities with Bank of Spain approval for their internal models for the purposes of calculating capital requirements for credit risk.
9“Standard portfolio” is deemed to be a hypothetical portfolio whose composition and characteristics would be similar to those of a standard portfolio at various Spanish financial institutions.
Executive Summary

This study provides different estimates of credit risk capital requirements under different scenarios and assumptions for the purpose of assessing the impact of such scenarios on both the regulatory capital model and economic capital consumption levels.

In addition to regulatory models, an economic capital model based on methodologies commonly applied by IRB institutions (hereinafter referred to as "the model" or "the economic capital model") has been used in order to measure the impact of the different scenarios. These methodologies are detailed in section 4 of the document (Methodology).

The scenarios and assumptions applied reflect different quantitative aspects that may have an impact on capital estimates and could therefore be considered by entities in their capital measurement and planning process.

The study is based on standard portfolios intended to be representative of the Mortgage and SME segments in Spain. The results obtained, even when conditioned by the portfolios selected, whose main features are outlined in section 3 (Information used) should be interpreted in general terms.

The following is a summary of the various assumptions and scenarios used in the study, applied to the Baseline Scenario or reference calculations for the purposes of obtaining a first comparison between regulatory and economic estimates.

The Baseline Scenario was also used in the analysis of capital requirement sensitivity to changes in confidence level (solvency level).

Based on this scenario, the following analyses were performed to measure the sensitivity of capital models to the economic cycle:

- Through-the-cycle adjustments: capital estimates are based on average probability of default estimates for an exposure and client cycle. The analysis measures the impact of considering deviations when estimating the average PD of portfolios over a full economic cycle under different assumptions.
- Procyclicality: for some entities’ internal models, a change in the economic cycle has resulted in an increase in capital consumption which exceeds that derived from a real deterioration in credit quality (effect known as “procyclicality”). The analysis performed aims to measure the impact on capital consumption of some of the proposals designed to mitigate this effect.

\[\text{10}^{\text{In any case, each entity should use its portfolio and models to obtain the results relating to their specific reality.}}\]

\[\text{11}^{\text{This analysis could be extended to other risk parameters, while for simplicity reasons it has been limited to Probability of Default.}}\]

![Graph](image-url)
In addition, several studies were conducted on the sensitivity of capital models to certain non-parameterizable aspects under the Pillar 1 regulatory model:

- **Stochastic LGD:** For regulatory capital calculation purposes, a stressed or downturn LGD is used which represents loss at the worst time within the cycle. The analysis measures the impact of considering that Severity, unlike what the regulatory model assumes, is not static or independent from the Probability of Default. Specifically, the analysis considers LGD volatility in addition to the correlation between PD and LGD (independent under the regulatory model).

- **Rating migration:** The loss due to credit risk (and its related capital charge) does not only result from default, but also from an impairment in the credit quality of assets. The analysis measures the impact on capital consumption of considering migrations between rating levels as an extension to the regulatory default/non default model.\(^1\)

- **Concentration:** The level of concentration has a direct impact on capital consumption, which increases in concentrated portfolios and decreases in diversified portfolios. However, whilst this is a fundamental issue in credit risk management, it is not reflected in Pillar 1 requirements, being therefore advisable to estimate its impact on capital. Specifically, name, exposure, sector, geographic, and risk segment concentrations are analyzed.

Finally, the analysis considers a last scenario, referred to as General Scenario, whose purpose is to consolidate the various individual scenarios. Thus, a conclusion can be reached as to the adequacy of the regulatory capital model under the different scenarios and assumptions considered.

In line with the above, capital requirements were calculated under the regulatory Standardized and Advanced IRB models\(^1\), and under the economic capital model, by applying the scenarios and assumptions described. The results obtained are detailed in section 5 of this document (Studies conducted and Analysis of findings).

The following are the main conclusions from the study, which differentiates between the IRB and the Standard approaches for comparison purposes:

**Advanced IRB method (Pillar 1)**

Overall, and except for certain PD and LGD correlation assumptions, outcomes obtained for the General Scenario and for each individually analyzed scenario all have economic capital requirements aligned with the minimum capital requirements estimated using the IRB method for the portfolios analyzed.

Nevertheless, regulatory Pillar 1 capital requirements show a high sensitivity to the specific assumptions used\(^2\), such as those relating to the through-the-cycle adjustment or degree of procyclicality of internal models. Likewise, economic capital requirements show a high sensitivity to critical aspects in their different dimensions (name, sector, geographic and risk segment).

A number of aspects from each scenario are highlighted below:

**Baseline scenario**

Regulatory capital estimates are very similar to those obtained using the economic capital model. Therefore, it is concluded that regulatory capital constitutes a correct approximation to loss distribution at the 99.9% regulatory confidence level.

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\(^1\)This effect is approached in the regulatory SME segment model using the maturity adjustment included in the capital formula, so that capital consumption is higher for longer-term exposures (the rating migration risk increases).

\(^2\)In contrast, for the purposes of Pillar 2, a simplified option for the calculation of sector and name concentration is included. In addition, entities could consider geographic concentration and concentration by risk segment within the general option (internal model).

\(^3\)Regulations on capital requirements (CRD 2007) allow banks to choose between two methods to calculate their capital requirements for credit risk. One method is to measure this risk in a standardized manner (Standard Method), based on external credit ratings and/or weights established by the supervisor, whereas the alternative method, subject to the supervisor’s explicit approval, allows banks to use their own internal rating systems for credit risk (IRB Approach). The IRB Approach comprises both the Foundation Approach and the Advanced Approach, whose main difference lies in the fact that internal estimates under the Foundation Approach are limited to PD, whereas EAD, LGD and M are also estimated under the Advanced Approach.

\(^4\)In this regard, it would be desirable to evolve towards greater standardization of certain industry assumptions in order to achieve more consistent end results.
However, it can be observed that an increase in the confidence level from 99.9% (as stipulated by the regulator) to 99.97% (approximately equivalent to a “AA” solvency level), would result in a 20% increase in capital consumption.

**Through-the-cycle adjustments**

The study shows that capital requirements are highly sensitive to through-the-cycle adjustments in parameters.

The impact that a change in through-the-cycle adjusted PD has on the baseline scenario can be illustrated from the fact that 25% variation in the average PD for the cycle translates into a 9% change in capital⁴. It should be noted that a 25% variation in the average default rate, which appears to be high, is in fact lower than the average default rate variation for the Spanish Financial System in the periods 1990-2009 and 1996-2009, where the rate changed from 3.19% to 1.98%¹. In other words, methodology decisions, as in this case the definition of the economic cycle, can have a significant impact on risk parameters, and thus on the calculation of capital⁵.

Despite the sensitivity of measurements in relation to the different methodological adjustments, there is currently no fully standardized calculation methodology across institutions. Some of the issues that make such standardization difficult are the quality of information, the length of historical data series, the historical variations in asset characteristics and the through-the-cycle adjustment assumptions (economic cycle definition⁶, average parameter calculation methodology, etc.).

**Pro cyclicality**

The effect of procyclicality as an alteration of risk parameters, especially PD, has a direct effect on the calculation of capital. However, its impact will differ between institutions, since it depends directly on their individual internal model characteristics. Thus, models built using short data series, or where payment behavior variables have a significant weight on the final score, tend to overestimate the impact on capital parameters, and thus on the calculation of capital⁷.

The results of the analyses performed reveal a large and disparate impact on capital requirements, depending on the different methodological and regulatory approaches used to address procyclicality. Thus, our findings for the two alternatives analyzed are as follows:

- Capital requirements would increase by 50% under the proposal to use a stressed PD instead of a through-the-cycle estimate of PD.
- Conversely, capital requirements would be reduced if the procyclicality effect were to be removed by approximating parameters through the use of historical averages⁸.

**Stochastic LGD**

As mentioned before, a stressed LGD is used in the regulatory model estimates. The proposed scenarios for analysis consider a through-the-cycle estimate of LGD which, as it is the case with PD, has an associated volatility (stochastic in nature) which is also PD correlated, i.e. in a period of increasing default rates, Loss Given Default will also increase.

In line with the above, the study concludes that the use of a stressed LGD in the regulatory model suffices to compensate for LGD volatility. Conversely, in the event that it should be deemed necessary to consider the effect of PD/LGD correlation in addition to volatility, capital requirements would be higher than they are at present. Such an impact would be greater on SMEs due to the higher volatility of risk parameters for this segment (PD, EAD and LGD), and to more concentrated exposures which magnify the impact of PD/LGD correlation. Specifically, the study reflects a 17% increase in the capital requirements for Mortgages and 40% for SMEs.

**Rating migration**

According to the results of the study, the impact of the rating migration on capital requirements is reflected in regulatory SME portfolio estimates through the maturity adjustment impact contained in the regulatory capital formula.

In contrast, for the Mortgage portfolio it cannot be ensured that the migration effect will be reflected in the regulatory model under certain stress conditions.

It should be noted that, because of how correlation between mortgage assets⁹ is calculated, the loss arising from rating migration is assumed to be reflected in the regulatory correlation measure, estimated at 15%, and thus assumed to be higher than the actual correlation value. Estimates made on the basis of default data from the Spanish Financial System revealed a correlation of 14.22% for Mortgages, hence not much lower than the regulatory measure.

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¹ Probability of default does not have a linear impact on capital.
² Source: Bank of Spain.
³ Any adjustments to LGD also have an impact on capital since, as it is the case with PD, an adjustment is necessary to obtain the downturn LGD used in regulatory estimates. It should be noted that the impact of LGD variations on regulatory capital is linear under the capital formula.
⁴ With regard to cycle definition, it is currently standard practice in the Spanish Financial System to consider the period from 1991 to date.
⁵ This reduction will depend on the impact of procyclicality on each model, which varies greatly and is largely dependent on the parameter estimation methodology used by each entity. To illustrate this, if the procyclicality effect on the Mortgage portfolio resulted in a 10% PD increase, then capital savings of 4% could be achieved by eliminating such 10% increase in PD. See “Explanatory Note on the Basel II IRB Risk Weight Functions”, of July 2005, issued by the Basel Committee.
Also, by not considering the impact of migration directly, but through the correlation of assets, regulatory capital is not sensitive to changes in average portfolio maturity.

Concentration

The effects of credit portfolio concentration have a substantial impact on capital consumption, not covered by Pillar 1 requirements. It is therefore essential to take them into consideration for the purposes of the internal capital assessment process under Pillar 2.

Considering concentration has a different impact on each entity. While it substantially increases capital requirements for entities whose portfolios are highly concentrated, it decreases these requirements for those with diversified portfolios (as in the case of geographical diversification of multinational entities).

The impact by concentration type: name, industry sector, geography and risk segment, is described below.

Name concentration

Name concentration arises from an entity’s larger client exposures, and it becomes more relevant in Large SME portfolios. The simulations performed showed a 20% increase in capital consumption over regulatory capital requirements, assuming the average concentration levels observed in these portfolios.

Sector concentration

Correlation estimates can differ significantly across industry sectors, as can be observed from estimates based on Spanish Financial System data. In contrast, regulatory correlation values do not depend on industry but on PD and counterparty size. As a result, there is no a priori guarantee that regulatory estimates of capital will adequately reflect sector correlation.

However, it can be concluded from the analysis that, overall, regulatory capital requirements are adequate for a portfolio of companies whose sector distribution is similar to that of the Spanish Financial System. Nevertheless, since the relative contribution by sector varies from one sector to another, this might not be the case for portfolios with a structure that differs significantly from that of the above mentioned system, particularly for entities with higher concentration levels in the real estate development and construction sectors.

Geographic concentration

The study undertaken analyzes geographic concentration for the Mortgage portfolio (which is more local in nature compared to the SME portfolio), and reveals that regional concentration can have a penalizing effect and does not affect all regions equally, resulting in relative differences in capital consumption of 50% or even higher (assuming that the entire portfolio is concentrated within a single region).

Concentration by risk segment

Lastly, the unequal distribution of the loan portfolio across the various risk segments can also have a significant impact on capital requirements under the economic model. It should be noted that the regulatory model considers segments independently, regardless of their individual risk concentration and diversification characteristics.

The impact of diversification on the Mortgage and SME portfolios reduces economic capital requirements by approximately 5% under the correlation assumptions used. However such impact could be substantially different if additional portfolios were included or their representativeness varied.

Standard Method (Pillar 1)

Overall, the capital requirement figures obtained under the Standard Approach are significantly more conservative than those obtained under the IRB Approach, and even higher than the values resulting from increasing the confidence level from 99.9% to 99.97%.

This means that, for most situations analyzed, the capital requirement estimated according to the Standard Method adequately covers all the different scenarios and assumptions used, excluding the effects of stochastic LGD and concentration.

However, the lack of risk parameters under this approach, as compared with the IRB method, makes the capital requirement figure much less sensitive to the credit quality of the portfolio or to the specific phase of the economic cycle. This could be reflected by the periodic updating of risk weights by the regulator in the Standard Approach.

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22For the purposes of this study, a Large SME is an enterprise whose turnover exceeds EUR 100 million.

23In particular, the assumption under which both portfolios are considered to be completely independent from each other was analyzed, resulting in a 20% decrease impact on capital.
The empirical analyses are based on two hypothetical portfolios corresponding to Mortgages and SMEs\(^{16}\), geographically distributed and default-free, with SMEs representing 51.85% of total exposure and Mortgages representing 48.15%.

The portfolios were built from public information\(^2\), Bank of Spain data, industry analyses and Management Solutions’ expertise and industry knowledge. Thus, the data relating to risk parameters were obtained through the assumption of calibrated distributions in order to elaborate portfolios that, for the purposes of this study, were representative of the Spanish Financial System within these segments.

The results obtained from the tests performed are intended to be compared with or extrapolated to those that would be obtained using real Mortgage and SME portfolios from Spanish entities.

Figure 1 shows a summary table containing statistics on the main risk parameters for the Mortgage portfolio used in the analyses\(^\text{26}\).

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\(^{16}\) The SME portfolio comprises loans to clients who are Legal Entities with a turnover below EUR 100 million.

\(^{2}\) E.g. Information of Prudential Relevance (Pillar 3).

\(^{26}\) The following parameters are shown for each portfolio, TtC PD: Through-the-cycle adjusted probability; EAD: Exposure at Default estimated in euros; Long Run (LR) LGD: average severity of the cycle; Downturn (DT) LGD: Severity at a time of stress; Time to Maturity; and, in the case of Mortgages, also Loan to Value (LTV), which is the ratio of the value of the loan to the price of the collateral.

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**Figure 1: Statistical summary of Mortgage portfolio**

<table>
<thead>
<tr>
<th>PD</th>
<th>EAD</th>
<th>LR LGD</th>
<th>DT LGD</th>
<th>Time to Maturity</th>
<th>LTV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.45%</td>
<td>96,000</td>
<td>8.66%</td>
<td>12.95%</td>
<td>16.12</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>11.20%</td>
<td>102,285</td>
<td>1.00%</td>
<td>2.83%</td>
<td>9.21</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.03%</td>
<td>100</td>
<td>7.50%</td>
<td>10.00%</td>
<td>0.01</td>
</tr>
<tr>
<td>Percentile 5</td>
<td>0.15%</td>
<td>8,000</td>
<td>7.50%</td>
<td>10.00%</td>
<td>0.25</td>
</tr>
<tr>
<td>Percentile 25</td>
<td>0.55%</td>
<td>35,000</td>
<td>8.00%</td>
<td>11.00%</td>
<td>9.00</td>
</tr>
<tr>
<td>Median</td>
<td>1.50%</td>
<td>70,000</td>
<td>8.50%</td>
<td>13.00%</td>
<td>15.00</td>
</tr>
<tr>
<td>Percentile 75</td>
<td>2.80%</td>
<td>130,000</td>
<td>8.50%</td>
<td>13.00%</td>
<td>23.50</td>
</tr>
<tr>
<td>Percentile 95</td>
<td>20.00%</td>
<td>250,000</td>
<td>11.50%</td>
<td>18.00%</td>
<td>32.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>60.00%</td>
<td>2,200,000</td>
<td>15.00%</td>
<td>23.00%</td>
<td>40.00</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>4.55</td>
<td>5.25</td>
<td>2.51</td>
<td>1.33</td>
<td>0.23</td>
</tr>
<tr>
<td>Curtosis</td>
<td>23.92</td>
<td>66.29</td>
<td>8.43</td>
<td>5.05</td>
<td>2.13</td>
</tr>
</tbody>
</table>
The time to maturity of the Mortgage portfolio appears to be high, while risk parameters such as PD or LGD are relatively low. Also, portfolio concentration, measured by means of the HHI index\textsuperscript{27} is 0.02%.

There is a concentration of exposures with LTV close to 80%, which means that, while LTV is 79.50% in percentile 75, it is does not exceed 80%\textsuperscript{28} until percentile 84, and only 16% of portfolios have LTV above 80%.

Expected Loss for the Mortgage portfolio is 0.64%.

The SME portfolio used in the analyses includes different products (loans, cards, credit lines, etc.) and its HHI concentration is 0.17%, higher than that for the Retail portfolio.

Figure 2 shows a summary table containing statistics on the main risk parameters of the SME portfolio used.

<table>
<thead>
<tr>
<th>PD</th>
<th>EAD</th>
<th>LR LGD</th>
<th>DT LGD</th>
<th>Time to Maturity</th>
<th>Turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.25%</td>
<td>440,000</td>
<td>28.45%</td>
<td>41.35%</td>
<td>1.93</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>11.42%</td>
<td>1,773,906</td>
<td>7.44%</td>
<td>10.82%</td>
<td>2.71</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.03%</td>
<td>100</td>
<td>15.00%</td>
<td>22.00%</td>
<td>0.01</td>
</tr>
<tr>
<td>Percentile 5</td>
<td>0.07%</td>
<td>2,000</td>
<td>16.00%</td>
<td>25.00%</td>
<td>0.07</td>
</tr>
<tr>
<td>Percentile 25</td>
<td>0.70%</td>
<td>25,000</td>
<td>21.00%</td>
<td>30.00%</td>
<td>0.30</td>
</tr>
<tr>
<td>Median</td>
<td>1.55%</td>
<td>90,000</td>
<td>32.00%</td>
<td>45.00%</td>
<td>0.90</td>
</tr>
<tr>
<td>Percentile 75</td>
<td>5.30%</td>
<td>250,000</td>
<td>32.00%</td>
<td>45.00%</td>
<td>2.50</td>
</tr>
<tr>
<td>Percentile 95</td>
<td>20.00%</td>
<td>1,800,000</td>
<td>40.00%</td>
<td>60.00%</td>
<td>5.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>70.00%</td>
<td>45,000,000</td>
<td>40.00%</td>
<td>60.00%</td>
<td>32.00</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>5.15</td>
<td>13,55</td>
<td>-0.13</td>
<td>-0.13</td>
<td>3.81</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>35.27</td>
<td>258,49</td>
<td>2.05</td>
<td>2.05</td>
<td>27.38</td>
</tr>
</tbody>
</table>

\textsuperscript{27}Index to measure concentration, takes values between 0% (no concentration) and 100% (maximum concentration). See Herfindahl index in the Methodology section.

\textsuperscript{28}Asset value, and therefore LTV, are of particular interest in the calculation of capital requirements under the Standard Approach, where the amount of the exposure above 80% of the value of the asset is given a higher risk weight, moving from 35% to 100% and even 150% for the amount of the exposure exceeding 95% of asset value.

The SME portfolio has a short time to maturity and, as can be expected, risk parameters above those for the Mortgage portfolio. Its Expected Loss is 2.51%.

The SME portfolio exposure is classified into eight industry sectors in a similar proportion to that of the Spanish Financial System, which is shown below:

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Methodology

Introduction

In order to perform a comparative analysis between the regulatory model and the economic capital model discussed above, it was necessary to further explore various methodological aspects with the aim of quantifying the scenarios and assumptions established.

In order to make this study more transparent, the economic capital credit risk model defined was based on commonly used methods.

As in the classical loss model, calculation of capital is based on the PD, EAD and LGD risk parameters. In principle, these parameters exhibit stochastic behavior, whose randomness may be due to specific and systemic factors.

This section details the most relevant methodological aspects considered in the calculation of loss distribution for the tests performed. These aspects are as follows:

- Base model: an initial model is defined, based on which different scenarios and assumptions are considered for the purpose of quantifying their impact on capital.

Loss distribution is generated through Monte Carlo simulation of 1 million stochastically generated default/non default scenarios, with probability of default as the expected value. The materialization of default depends both on systemic factors (state of the economy) and on idiosyncratic factors.
The stability of model results is validated by means of a numerical test.

- **Stochastic severity**: definition of the methodology to be used for considering LGD volatility and PD and LGD correlation.

- **Rating migration**: definition of the methodology to be used for including the effect of rating migrations over one year in order to capture the loss arising from the credit impairment of exposures with maturities of more than one year.

- **Measuring concentration**:
  - The Herfindahl Index (HHI), the statistical measure used to determine the concentration level of a credit unit, is defined.
  - The methodology for estimating correlation between credit assets is defined.

These methodological aspects are described below:

**Base model**

Loss distribution for the base model is calculated using the Monte Carlo method and is based on a Vasicek-type approach to default, and on the consideration that severity is constant and equal to downturn LGD.

The Vasicek model defines an expression for PD which is conditioned by a systemic scenario and derived from the Merton approach. Thus, a counterparty’s ability to pay is represented as a random variable $v$ with Normal distribution $N(0,1)$. Counterparty default ($I_i = 1$) is observed when this ability to pay falls below a threshold which is a function of the TtC PD of the counterparty.

Variable $v$ can be decomposed into a systemic factor $z$ common to all counterparties and a specific factor $y_i$ which varies by counterparty, so that to achieve this:

$$I_i = 1 \text{ when } v < N^{-1}(pd_i)$$

By conditioning this expression to the value of the systemic factor and finding the value of the specific factor, the Vasicek formula is obtained for counterparty default $i$.

$$u_i < pd_i^* = N\left(\frac{N^{-1}(pd_i) - \sqrt{\rho_i} z}{\sqrt{1 - \rho_i}}\right)$$

Where:

- $pd_i$: TtC PD of counterparty $i$.
- $\rho_i$: Intraunit correlation of counterparty $i$. Regulatory correlation is used in the base model.
- $z$: $N(0,1)$ distributed systemic factor (common to all counterparties in a given portfolio).
- $u_i$: Specific factor for counterparty $i$, uniformly distributed over the interval $[0,1]$.

The equation (1) defines a random Bernoulli variable with parameter $pd_i^*$. That is, default is a dichotomic variable with value 0 or 1. The frequency with which 1 results is given by the $pd_i^*$ parameter.

Given the above, the Monte Carlo loss distribution for the base calculation can be expressed as:

$$L = \sum_{i=1}^{N} \text{Ber}(pd_i^*) \cdot \text{ead}_i \cdot \text{lgd}_i$$

Where:

- $N$: Number of counterparties for the portfolio.
- $\text{ead}_i$: EAD for each counterparty.
- $\text{lgd}_i$: DT LGD for each counterparty.

Therefore this model calculates loss distribution consistently with the IRB approach, whilst taking it a step further by considering default for each counterparty, without assuming infinite granularity.

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As in the expression that determines the amount of capital under the IRB approach.
The stability of the results obtained using the base model is analyzed, identifying the error that results when the loss distribution for the corresponding portfolio is simulated. The base model is run 10 times on both portfolios and capital is calculated for the two confidence levels considered. The volatility measures for the values obtained are subsequently analyzed.

**Analysis and conclusions**

The data obtained from the stability test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Mortgages</th>
<th>SMEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenarios at</td>
<td>99.9%</td>
<td>99.97%</td>
</tr>
<tr>
<td>1</td>
<td>2.05%</td>
<td>2.47%</td>
</tr>
<tr>
<td>2</td>
<td>2.04%</td>
<td>2.48%</td>
</tr>
<tr>
<td>3</td>
<td>2.04%</td>
<td>2.47%</td>
</tr>
<tr>
<td>4</td>
<td>2.04%</td>
<td>2.50%</td>
</tr>
<tr>
<td>5</td>
<td>2.02%</td>
<td>2.44%</td>
</tr>
<tr>
<td>6</td>
<td>2.04%</td>
<td>2.48%</td>
</tr>
<tr>
<td>7</td>
<td>2.03%</td>
<td>2.49%</td>
</tr>
<tr>
<td>8</td>
<td>2.02%</td>
<td>2.48%</td>
</tr>
<tr>
<td>9</td>
<td>2.02%</td>
<td>2.47%</td>
</tr>
<tr>
<td>10</td>
<td>2.04%</td>
<td>2.49%</td>
</tr>
<tr>
<td>Mean</td>
<td>2.03%</td>
<td>2.48%</td>
</tr>
<tr>
<td>Max-Min</td>
<td>0.02%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.43%</td>
<td>0.62%</td>
</tr>
</tbody>
</table>

Based on loss distribution, capital at confidence level \( \alpha \) is calculated simply by subtracting expected loss from the percentile for the level being considered:

\[
\text{Capital}^\alpha(l) = \text{Percentile}^\alpha(l) - \text{Mean}(l)
\]

**Monte Carlo simulation**

Loss distribution is obtained by Monte Carlo simulation in all cases. The general scheme for calculation is as follows:

1. Simulation of systemic factors.
2. Parameter adaptation to the systemic scenario.
3. Computation of the loss for each counterparty under the defined scenario.
4. Obtainment of the loss for the total unit under the defined scenario.
5. Iteration of the above process for all simulations considered.

**Monte Carlo stability test**

Despite being a powerful tool, Monte Carlo simulation has an inherent variability, since it reproduces the behavior of variables that are not deterministic. It is therefore important to validate that the proposed number of simulations leads to stable results.

Despite being a powerful tool, Monte Carlo simulation has an inherent variability, since it reproduces the behavior of variables that are not deterministic. It is therefore important to validate that the proposed number of simulations leads to stable results.

**CAPITAL ADEQUACY FOR CREDIT RISK: A PRACTICAL EXERCISE**

(5)

All simulations will be performed using 1,000,000 scenarios.
As can be observed, performing 1,000,000 simulations is sufficient to ensure the stability of the Monte Carlo simulation, the maximum difference in capital charges between simulations being 0.14% for SMEs provided that capital is calculated at a 99.97% confidence level. These differences, due to the randomness of the simulation, are not significant and do not affect test results.

**Stochastic severity**

Loss distribution calculation with stochastic severity is an extension of the base model so that counterparty severity, which was assumed to be constant, is now a random variable (r.v.).

The stochastic LGD for each counterparty is defined as:

\[
\text{LGD}_{i}^{*} = N\left(\frac{N^{-1}\left(\text{LGD}_{i}\right) - \sqrt{\alpha} w}{\sqrt{1-\alpha}}\right)
\]

(6)

Where:

- \(\text{LGD}_{i}\) LR LGD for counterparty \(i\).
- \(\alpha\) LGD sensitivity to the systemic component for each counterparty (assumed to be constant for all counterparties). It holds that: 0 < \(\alpha\) < 1.
- \(w\) Systemic factor with an impact on LGD. It is common to all counterparties and is distributed as \(N(0,1)\).

In addition, \(z\) (systemic PD factor) and \(w\) (systemic LGD factor) have correlation \(k\).

This stochastic approach to LGD assumes the same functional relationship between the parameter and the systemic factor as in the case of the PD model, and allows for inclusion of correlation between both parameters from the \(k\) coefficient. Whilst the literature provides other alternatives for modeling random severity, Beta distribution being the most widespread, the chosen approach has two significant advantages:

- Allows for setting the mean distribution at the Long Run LGD.
- Facilitates inclusion of the correlation between PD and LGD based on the correlation between the \(z\) and \(w\) factors.

Furthermore, its shape is similar to that of the Beta distribution, as can be seen from Figure 6, where the Vasicek and Beta distributions have been adjusted for the same set of data.

Parameters \(\alpha\) and \(k\) can be estimated from historical LGD series (for \(\alpha\)) and from historical LGD and PD series (for \(k\)).

From this approach to stochastic LGD, the loss distribution for a particular portfolio will be determined by:

\[
L = \sum_{i=1}^{N} \text{Ber}(pd_{i}^{z})ead_{i}\text{LGD}_{i}^{*}
\]

(7)

**Figure 6**: Vasicek and Beta distributions fitted to the same data set
**Rating migration**

Under the previous approaches, the loss distribution reflected default over a one year period, so that the risk being considered was that of counterparty default over this period of time.

Under a market value approach\(^1\), the loss distribution reflects the loss relating to the change in the value of a counterparty as a result of a deterioration in its credit quality over a one year time horizon.

Thus, if \(V(i,r)\) is the value of all positions of counterparty \(i\), and its rating (credit quality) is \(r\), then the loss associated with such counterparty over a one year time horizon will be given by:

\[
L_i = V(i,r_f) - V(i,r_i)
\]

\((8)\)

Where:

- \(i\) Positions with counterparty \(i\) at the end of the one-year period.
- \(r_f\) Rating at the end of the one-year period.
- \(r_i\) Rating at the beginning of the one-year period.

The transition from the initial to the final rating is given by a migration matrix \(M\), whose size is \(s\) (\(s\) being the number of possible ratings including that for default). \(M(i, j)\) is the probability of migrating from rating \(i\) to rating \(j\). It is reasonable to assume that these probabilities change throughout the business cycle, even when ratings are based on \(T\&T\) measures, therefore it is advisable to include systemic influence in \(M\).

Since the loss for each counterparty has been obtained according to \((8)\), total portfolio loss is given by:

\[
L = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} [V(i,r_f) - V(i,r_i)]
\]

\((9)\)

Therefore, to obtain the loss distribution using a market value approach it is necessary to define:

- A \(V(i,r)\) function that will make it possible to calculate the value of all positions for each counterparty on the basis of their rating.
- A migration matrix \(M\) that will also include the systemic influence.

The proposed \(V(i,r)\) function is defined based on the term of the transaction and its EAD, as well as a repayment scheme where periodic payments are made\(^2\).

With regard to the inclusion of systemic influence in the migration matrix, a conditioned approach has been assumed so that the probability of migration is perturbed depending on the systemic scenario, the probability of migration to a better rating increasing under a positive scenario and decreasing under a negative scenario, whilst the sum of the probabilities in each row of the transition matrix always remains 1.

\(^1\)The market value approach is incorporated in various commercial solutions such as Creditmetrics and CreditPortfolioView.

\(^2\)In accordance with the term of each transaction, any payments remaining to maturity are computed by discounting each payment to present value using a spread determined by the final rating.
Measuring concentration

Herfindahl-Hirschman index (HHI)

Concentration is measured using the Herfindahl-Hirschman index (HHI), defined as:

$$HHI = \frac{\sum_{i} (ead_i)^2}{\left(\sum_{i} ead_i\right)^2}$$  \hspace{1cm} (10)

This definition can be used to verify that, for all portfolios with infinite granularity $HHI = 1/N$, where $N$ is the number of counterparties in each unit, whilst in the event that a counterparty accumulated the entire exposure, $HHI$ would equal 1 (100% in percent terms).

The usefulness of the $HHI$ index lies in its comparison of the degree of concentration of different portfolios, so that the greater the concentration the higher the values of this index, which has a maximum value of 1.

For reference purposes, the private Mortgage portfolio that has been used for this analysis has a $HHI$ of 0.02%, the SME portfolio has a $HHI$ of 0.17%, and the typical $HHI$ for a Large SME portfolio can be close to 3.5%, although the $HHI$ for the portfolio of largest Companies can have a certain level of volatility in line with the number of large positions held at each particular time.

Asset correlation

Correlation within a portfolio (intraunit correlation)

In order to calculate the correlation of a portfolio, default probabilities $pdz(t)$ have been approximated, given a specific state of the economy $z(t)$, by the annualized default rates of portfolio $\tilde{t}$:

$$t_i \approx pdz(t) = N \left( \frac{\sqrt{pdz(t)}}{\sqrt{1 - \rho}} \right)$$  \hspace{1cm} (11)

It is possible to obtain $\rho$ from the above expression:

$$\rho = \frac{\text{Var}(N^{-1}(t_i))_{r \in \mathbb{R}}}{\text{Var}(N^{-1}(t_i))_{r \in \mathbb{R}}}$$  \hspace{1cm} (12)

Correlation between different portfolios (interunit correlation)

In order to obtain aggregated capital for various portfolios, a factor correlation approach was selected. Thus, the loss distribution for $j$ is obtained, as described in (4) as:

$$L_j = \sum_{i=1}^{N_j} \text{Ber}(pdz(t_i))ead_i$$

Where $z(t_i)$ is the systemic factor for portfolio $j$ and follows a Normal distribution $N(0,1)$. The matrix of correlation between factors $z(t_i)$ is given by $\Omega$ and can be calibrated using historical default series.

This approach makes it possible to incorporate interunit correlation naturally from factor structure.

Figure 7: Portfolio engineering for the calculation of aggregate economic capital

**Note:** Used by the Bank of Spain to estimate the additional capital charge for name and sector concentration under the Capital Self-Evaluation Process framework. To date, it is common practice to estimate these correlations by means of historical reconstruction (using macroeconomic regression) of historical default rate series. For the sake of simplicity, in this analysis correlations are estimated empirically through historical default values relating to the Mortgage and SME portfolios.
This section contains the detail of the capital consumption analyses performed, as well as the conclusions derived from them. These analyses are based on the economic capital model and the Mortgage and SME portfolios described earlier in the document.

The results obtained will be more or less representative of those that would be obtained for other portfolios depending on their similarity to the portfolios under study, although the outcome from the analyses performed may be useful regardless of the portfolios used. In any case, it would be of interest for entities to replicate some of these tests internally as part of their Capital Self-Evaluation Process (Pillar 2).

The cases contained in this section cover the following aspects:

- **Baseline scenario**: this is the reference calculation and is used, amongst other things, to make a first comparison between model and regulatory estimates. It also serves to measure the sensitivity of capital requirements to changes in the confidence level, and thus to the solvency level.

- **Through-the-cycle adjustment**: average through-the-cycle PD estimates are used in capital calculations. The analysis performed measures the impact of considering deviations in the average PD estimates for portfolios over a full economic cycle under different assumptions.

- **Procyclicality**: the effect of the change in cycle on the internal models of some entities has seen an increase in capital consumption exceeding that derived from the actual deterioration in credit quality (the so called “procyclicality effect”). The analysis performed intends to measure the impact on capital consumption of some of the current proposals aimed at mitigating such an impact.

- **Stochastic LGD**: a stressed or downturn LGD representing the loss at the worst time in the cycle is used in the calculation of capital. The analysis performed measures the impact of considering that Severity, unlike in the regulatory model, is neither static nor independent from the Probability of Default. In particular, the study considers LGD volatility and the correlation between PD and LGD (independent under the regulatory model).
It can be seen that Monte Carlo simulation at a 99.9% confidence level results in capital consumption values in line with the advanced IRB approach, whilst it is in Mortgages where greater similarities between both estimates occur. As for SMEs, the reason why capital requirements under the IRB approach are higher than those under Monte Carlo at a 99.9% confidence level is because a maturity adjustment is applied to SMEs and not to Mortgages under the advanced IRB approach. In fact, using the advanced IRB method to calculate capital for SMEs without adjusting for maturity results in a 5.49% capital charge, similar to the 5.61% value obtained under Monte Carlo, albeit slightly lower since Monte Carlo takes the SME portfolio concentration into consideration.

This does not occur for Mortgages where, as mentioned before, the outcome is basically the same under advanced IRB as under Monte Carlo simulation at a 99.9% confidence level, due to the almost infinite granularity of this portfolio, which in theory ensures convergence between both methods.

It can be seen that Monte Carlo simulation at a 99.9% confidence level results in capital consumption values in line with the advanced IRB approach, whilst it is in Mortgages where greater similarities between both estimates occur. As for SMEs, the reason why capital requirements under the IRB approach are higher than those under Monte Carlo at a 99.9% confidence level is because a maturity adjustment is applied to SMEs and not to Mortgages under the advanced IRB approach. In fact, using the advanced IRB method to calculate capital for SMEs without adjusting for maturity results in a 5.49% capital charge, similar to the 5.61% value obtained under Monte Carlo, albeit slightly lower since Monte Carlo takes the SME portfolio concentration into consideration.

Following is a description of each of the analyses performed.

**Baseline scenario**

**Overview and objectives**

Regulatory values obtained under the IRB and standardized approaches are compared with those obtained using the base model described earlier in this document. In the case of the IRB method, capital calculations are shown without considering the maturity effect in the regulatory calculations so that the various capital calculation models can be more easily compared.

The simulation used in the base model fully includes the name concentration risk of the portfolio, therefore greater differences are expected for the SME portfolio than for the Mortgage portfolio.

**Analysis and conclusions**

Figure 8 below details the capital charges by exposure obtained for the base test:
**Through-the-cycle adjustment**

**Overview and objectives**

Through-the-cycle adjustment methodologies contain implicit calculation errors. For example, no historical default rate series comprising a full economic cycle (according to the market standard that considers such cycle to have commenced in 1991\(^3\)) is available for the purposes of estimating average portfolio default. This makes it necessary to rebuild this series using different methodologies, which results in an implicit calculation error that might be significant in some cases.

In addition, methodological through-the-cycle adjustment issues such as the way in which average through-the-cycle default or central tendency are assigned by rating level, can increase error in the calculations.

In order to include the sensitivity of capital to this implicit uncertainty, it is proposed that the impact of modifying the through-the-cycle adjustment applied to the probability of default\(^4\) on capital requirement calculations should be analyzed.

Since the through-the-cycle adjustment defines the TtC PD level, the effect of having applied various through-the-cycle adjustments will be approximated by multiplying the PD by a multiplication coefficient. Thus, TtC PD for both portfolios is multiplied by 80%, 90%, 100% (in this case, the original value), 110% and 125%, and both regulatory and base model capital values are calculated for each of these adjustments.

**Analysis and conclusions**

The capital charges obtained are as follows:

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\(^3\)Thus, for instance, the average monthly default rate for the Spanish Financial System varies from 1.48% for the period from 2000 to 2009, to 1.98% for 1996-2009 and 3.19% for 1990-2009.

\(^4\)This could also be calculated for the LGD parameter, since calculations are cycle adjusted for both parameters.
these estimates under the Standard Approach are higher than those obtained using the IRB approach and the economic capital model even for the most severe scenarios.

In contrast to the Standard Approach, the IRB and Monte Carlo methods are affected by the through-the-cycle adjusted PD, the capital charge increasing roughly in line with TtC PD increases. For example, in both cases a 25% increase in PD results in a 9% increase in capital.

In general, it can be observed that sensitivity of capital to changes in the probability of default is not linear, with changes in the probability of default having an impact on capital that is lower than the change itself.

**Procyclicality**

**Overview and objectives**

Procyclicality is evident from the fact that, under the currently deteriorated economic conditions, models tend to classify counterparties into the worst rating levels, the PD of portfolios being too harshly penalized. This double deterioration, systemic in origin, results in excessive sensitivity to the economic cycle, which can in turn result in a sharp and sudden increase in the capital requirements imposed to institutions, with the related implications for their solvency and granting of credit.
In view of this, regulators are considering measures to mitigate model procyclicality. In particular, the Basel Committee is undertaking an analysis to measure the impact of two alternative methods:

- Using the highest average PD historically observed by the entity within each rating tranche, which can be understood to be a downturn PD, to calculate minimum capital requirements.

- Using an historical average PD for each rating tranche, in line with the FSA’s proposal.

This test aims to analyze the impact of one of the proposals, i.e. the use of the historically highest PD used by the entity for each type of exposure in order to approximate this downturn PD.

To this end, economic capital will be calculated using a stressed PD. This stressed PD is, according to the historical information available for 1991-2009, 2.86 times higher than the average PD observed for this period.

It can be seen that the use of this multiplicative adjustment results in a substantial increase in capital consumption - 50% for Mortgages and 45% for SMEs under Monte Carlo at 99.9%.

Therefore, implementing this measure, intended to achieve greater capital adequacy to provide more coverage in the event of adverse economic conditions such as the current situation, would result in a significant increase in capital consumption with respect to current capitalization standards.

These results are in contrast with those that would be obtained today by using an historical average PD for each rating level, whereby capital requirements would decrease with respect to current requirements, since what is intended in this case is only to reflect actual portfolio impairment whilst eliminating the effect of procyclicality, which at present increases PD, as previously described. Such variation in capital consumption depends on the procyclicality level in the models of each entity, which will vary according to model characteristics.

Analysis and conclusions

The capital charges obtained are as follows:

*Figure 13: Capital consumption by exposure with stressed PD*

<table>
<thead>
<tr>
<th></th>
<th>Basel II Standard</th>
<th>Basel II Advan. IRB</th>
<th>Basel II Advan.IRB without maturity</th>
<th>Monte Carlo 99.9%</th>
<th>Monte Carlo 99.97%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgages - Original</td>
<td>2.93%</td>
<td>2.03%</td>
<td>2.03%</td>
<td>2.02%</td>
<td>2.47%</td>
</tr>
<tr>
<td>Mortgages - Stressed PD</td>
<td>2.93%</td>
<td>3.03%</td>
<td>3.03%</td>
<td>3.02%</td>
<td>3.62%</td>
</tr>
<tr>
<td>Companies - Original</td>
<td>7.42%</td>
<td>6.75%</td>
<td>5.49%</td>
<td>5.61%</td>
<td>6.78%</td>
</tr>
<tr>
<td>Companies – Stressed PD</td>
<td>7.42%</td>
<td>9.28%</td>
<td>8.04%</td>
<td>8.16%</td>
<td>9.42%</td>
</tr>
</tbody>
</table>

*Section II. 4 of consultative document “Strengthening the resilience of the banking sector”, of 17 December 2009, by the Basel Committee.

See note “Variable Scalar Approaches to Estimating Through-the-cycle PDs” by FSA, February 2009. Specifically, a Segmentation Based Proposal (orthogonal segments) is described, which consists of performing various calibrations and adjustments to the acyclical parameters associated with each portfolio.

*Source: Bank of Spain.*
**Stochastic LGD**

**Overview and objectives**

The value of a loan’s guarantees can experience substantial changes from the time a client defaults. Therefore, it is advisable to know capital consumption under a stochastic LGD model, paying particular attention to two parameters: LGD volatility and its potential correlation to PD, since both parameters relate to the economic cycle.

Regulatory capital and capital obtained from the base model for both portfolios is compared to that obtained from the model including stochastic LGD with the parameters estimated from the historical series, using both the original parameter values and the same values increased/decreased by +15%/-15%, respectively, for the purpose of analyzing the impact on capital of LGD volatility and PD/LGD correlation.

Following is a description of LGD volatility (in absolute terms) and PD/LGD correlation parameters estimated from historical series for the defined portfolios:

*Figure 15: LGD volatility factors* and PD/LGD correlations used

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>LGD Volatility</th>
<th>PD/LGD Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgages</td>
<td>3.21%</td>
<td>25.28%</td>
</tr>
<tr>
<td>SMEs</td>
<td>8.35%</td>
<td>29.72%</td>
</tr>
</tbody>
</table>

Specifically, the following calculations are performed:

- For Mortgage and SME portfolios, the capital calculated is that associated with using the model with stochastic LGD while ignoring the potential PD/LGD correlation in order to isolate the effect of LGD variability on capital. Besides calculating volatility, estimates of capital are obtained by increasing/decreasing LGD volatility by +15% and -15%, respectively, for the purpose of showing the impact that an error in the calculation of this parameter can have on capital.

- Capital calculations for Mortgage and SME portfolios including both effects: LGD volatility and PD/LGD correlation. To this end, LGD volatility is set at its calculated value, whilst three values are considered for PD/LGD correlation: its calculated value and two variations of +15% and -15% respectively, for the purpose of showing the impact that an error in the calculation of PD/LGD correlation can have on capital.

**Analysis and conclusions**

The capital charges obtained are as follows:

*Figure 16: Capital consumption by exposure for analyses without PD/LGD correlation for Mortgages*

<table>
<thead>
<tr>
<th>Mortgages</th>
<th>Basel II</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>IRB</td>
<td>IRB with</td>
<td>LR LGD</td>
</tr>
<tr>
<td>99.9%</td>
<td>99.97%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>LGD</th>
<th>PD/LGD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>2.93%</td>
<td>20.3%</td>
</tr>
<tr>
<td>Volatility</td>
<td>2.93%</td>
<td>20.3%</td>
</tr>
<tr>
<td>Correlation</td>
<td>2.93%</td>
<td>20.3%</td>
</tr>
</tbody>
</table>

*Long Run LGD (average cycle severity), as opposed to Downturn LGD (stressed severity) is used as a basis when stochastic LGD estimates are made in this test.*

*This is consistent with a 95% confidence level for the estimated LGD distribution.*

*The role of the volatility factor in stochastic LGD is similar to that of the intraunit correlation in the base test.*
Including the effects of LGD volatility and PD/LGD correlation, as opposed to considering static LGD, results in increased capital. Increased volatility results in greater uncertainty regarding potential portfolio losses, and thus higher capital charges. The inclusion of a positive PD/LGD correlation magnifies losses under extreme scenarios in the sense that higher default scenarios result in greater losses by unit of exposure.

For Mortgages, it appears that the capital obtained when applying Downturn LGD is greater than that calculated by applying the LGD volatility effect alone. This means that the stressed LGD falls within a higher LGD distribution percentile than that which generates portfolio losses at a 99.9% confidence level.

In contrast, for SMEs the empirical volatility scenario produces capital consumption values similar to those resulting from the use of a stressed LGD, showing that Downturn LGD should be closer to the distribution percentile that results in losses at a 99.9% confidence level.

Once LGD is considered as a stochastic variable, it is necessary to include a PD/LGD correlation factor to determine how both magnitudes behave together on average under the simulated scenarios.

The inclusion of this effect has a very significant impact on capital consumption figures which, based on empirical volatility and correlation values, results in relative capital consumption increases of 17% for Mortgages (from 2.02% to 2.37%) and 40% for SMEs (from 5.61% to 7.83%).
It should be mentioned that, in the event that empirical correlation has the same magnitude in both portfolios, its impact on the SME portfolio is 50% higher than on the Mortgage portfolio, since it includes the effect of LGD volatility and PD/LGD correlation. This outcome is related to the fact that volatility is higher for the risk parameters in the SME portfolio (PD, EAD and LGD), which, as in the defined portfolio, can increase the impact when stochastic LGD is considered.

With regard to the LGD distribution and PD/LGD correlation sensitivity to changes in volatility, capital consumption figures for Mortgage and SME portfolios reveal similar sensitivity levels - around 8% - 9% for the range of values established.

**Rating migration**

*Overview and objectives*

The rating migration adjustment reflects the variation in the value of assets to changes in their credit quality. Some academic models available in the market are widely applied by leading entities that take this effect into account (e.g. Creditmetrics). Under regulatory models, rating migration is reflected by a maturity adjustment in the SME portfolio and by asset correlation in the Mortgage portfolio. In the case of SMEs, such an adjustment, in addition to considering the time to maturity of exposures, uses PD to reflect a possible deterioration in the credit quality and market value of counterparties.
The purpose of this test is to show the capital derived under a migration loss approach for the SME portfolio. An unconditional migration matrix has been used to calculate the rating migration impact using the economic capital model. This matrix was obtained from public Standard & Poors data and from market spreads.

With regard to the Mortgage portfolio, the impact of considering migration ratings has not been estimated for test simplicity purposes, and assuming that this impact is lower than for the SME portfolio⁴⁶.

However, in order to contrast the regulatory assumption that rating migration in the Mortgage portfolio is considered in the regulatory correlation, the correlation between mortgage assets has been estimated from aggregated default rate data for the Spanish Financial System, showing that this correlation is slightly lower than the regulatory value. Specifically, a 14.22% correlation was estimated, as opposed to the regulatory value of 15%. Therefore, under a stressed scenario, the impact of migration on the Mortgage portfolio must be taken into account, particularly when recent exposures represent a significant volume. In any case, the regulatory estimate would be more transparent if both effects, correlation and migration, were calculated separately.

⁴⁶From the fourth year onwards, the Mortgage default curve decreases, therefore rating migrations are, globally within the portfolio, always to better ratings.
Analysis and conclusions

The capital consumption figures obtained are presented below:

**Figure 22: Capital consumption by exposure considering the rating migration**

<table>
<thead>
<tr>
<th></th>
<th>Basel II</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Advanced</td>
</tr>
<tr>
<td></td>
<td>IRB</td>
<td>IRB without</td>
</tr>
<tr>
<td>SMEs:</td>
<td></td>
<td>maturity</td>
</tr>
<tr>
<td>without migration</td>
<td>7.42%</td>
<td>6.75%</td>
</tr>
<tr>
<td>with migration</td>
<td>7.42%</td>
<td>6.75%</td>
</tr>
</tbody>
</table>

When migration is considered, the capital charge calculated at a 99.9% confidence level using Monte Carlo (including migration) is in line with Advanced IRB.

Including losses associated with rating migration increases capital by around 19-21% in relative terms with respect to the case where only losses arising from counterparty default are considered. This increase would be greater for portfolios with longer-term exposures and better ratings. Specifically, the SME portfolio used has an average maturity of almost two years.

It should be noted that these estimates are highly sensitive to the migration matrix and spreads used. In this case, and as discussed before, average estimates for the cycle are used, whilst, under a stress scenario causing higher migration and wider spreads, the capital charge under the model could exceed regulatory capital requirements.

Concentration.
Name concentration

Overview and objectives

In certain portfolios, (e.g. Large SMEs, with turnover above EUR 100 million), a small number of counterparties accumulate a large portion of the portfolio exposure and can therefore have a large impact on capital consumption despite their reduced number. In order to take this impact into account, a test has been developed which reflects capital consumption behaviour for different concentration levels.

Capital is calculated, using the various models, for the original portfolio and for those obtained after exposure has been increased by a multiplicative coefficient applied to the 10 counterparties with the largest EAD. Subsequently, the exposure for all counterparties is re-scaled for the purpose of obtaining a portfolio whose exposure is equal to that of the original portfolio. The multiplication coefficients considered were 4.7 and 15, which produced HHI values of 0.83%, 1.66% and 3.52% respectively, from the initial 0.17% for the SME portfolio. These HHI values are similar to those obtained for a standard Large SMEs portfolio.

The BIS II curves for Corporates and SMEs were used to obtain results under the regulatory model.

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47 Eventual gains from counterparty migration to better rating levels were allowed under this test. However, the outcome that would result by allowing losses only, reducing profits to zero, is similar to that described in this document.

48 Comparative analyses performed by Management Solutions suggest that concentration could be even higher for some of the Large SME portfolios, therefore it would be advisable to undertake this calculation internally.
Analysis and conclusions

The following are the capital consumption values obtained:

\[
\text{Figure 24: Capital consumption by SME exposure for different concentration levels}
\]

<table>
<thead>
<tr>
<th></th>
<th>Basel II</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Advanced IRB</td>
<td>Advanced IRB</td>
</tr>
<tr>
<td></td>
<td>without maturity</td>
<td>Corp. curve</td>
</tr>
<tr>
<td>Original (HHI = 0.17%)</td>
<td>6.75%</td>
<td>5.49%</td>
</tr>
<tr>
<td>SMEs (HHI = 0.83%)</td>
<td>7.28%</td>
<td>5.69%</td>
</tr>
<tr>
<td>SMEs (HHI = 1.66%)</td>
<td>7.63%</td>
<td>5.82%</td>
</tr>
<tr>
<td>SMEs (HHI = 3.52%)</td>
<td>8.17%</td>
<td>6.02%</td>
</tr>
</tbody>
</table>

The initial HHI for the SME portfolio is 0.17%, higher than that for the Mortgage portfolio\(^9\). As exposure to the larger counterparties increases, this concentration coefficient increases (HHI can become 20 times higher), resulting in higher capital consumption, which increases by 55.20% and 48.61% using Monte Carlo at 99.9% and 99.97%, respectively. Therefore, it can be concluded that capital consumption is highly sensitive to exposure concentration.

In contrast, the IRB method is not sensitive to name concentration. However, in the test conducted, artificially modifying the level of concentration by increasing exposure to the 10 counterparties with the highest exposure resulted in exposure concentration involving counterparties with longer maturity positions and greater expected loss, which has increased capital requirements under the IRB approach. In order to isolate the effect of concentration, the impact of an increase in the average maturity of the portfolio is eliminated, comparing the IRB capital estimates without maturity adjustment (6.02%) against base model estimates (8.71%) at the point of maximum concentration, observing a concentration impact of 44%\(^10\).

Likewise, but under the assumption that the regulatory curve for SMEs is used, where concentration is partially reflected through higher intra-unit correlation, the impact of concentration is 17%.

**Sector and geographic concentration**

Sector and geographic concentration is measured through a portfolio’s internal correlation (intra-unit correlation). This correlation constitutes an indicator that reflects the dependency occurring between portfolio counterparties, so that, for a given state of the economy, the greater the intra-unit correlation the more similar the behavior of the counterparties towards each other in the event of default.

\(^9\)The HHI for Mortgages is 0.02%.
\(^10\)On the basis of the baseline scenario tests, it is assumed as a hypothesis that the impact resulting from an increase in expected loss is similar in both models.
Sector concentration

Overview and objectives

The aim of the test performed is to assess whether regulatory requirements adequately reflect sector concentration. For both the portfolio analyzed and the Spanish Financial System, business sectors showing the highest concentration are real estate development, construction and tourism. Therefore these are the sectors that should be analyzed in further detail.

Three possible correlations are considered in order to measure concentration:

- Correlation derived from Spanish Financial System (SFS) data, which, for SMEs, is available at the industry sector level.
- Correlation established in the Basel II regulatory capital formulas, which for SMEs depends on PD and company size.
- Correlation derived from the volatility of the historical default rate series built for the SME portfolio. This constitutes a single estimate for the entire SME segment since no disaggregated historical information is available by industry sector. In any case, this correlation is useful to emphasize the importance of estimating internal correlations which may differ from regulatory values or from those estimated on the basis of Spanish Financial System data.

The correlation estimates performed show substantial differences between correlations of certain sectors depending on the source of the estimate. This occurs as a result of using different drivers and data, and it should be emphasized that regulatory correlation does not take the industry sector into account, only the PD and company size.

Regulatory correlation values are similar for the real estate development and construction sectors, although slightly lower than those estimated based on Spanish Financial System data, whilst for the tourism sector regulatory correlation estimates are significantly higher. Furthermore, correlation values estimated using the defined portfolio are always higher than the rest of correlation values.

In the test, capital by industry sector is estimated, and the capital charge for each sector in the portfolio under analysis is calculated. In this regard, the capital consumed by each sector depends on the risk parameters for the companies within each particular sector, and on the correlation assigned. In any case, the SME portfolio is a standard portfolio built to be representative of a portfolio of companies within the Spanish Financial System, and aims to reflect the credit characteristics of each segment.

Calculations are also performed at the global level to analyze whether, in general terms, the regulatory model covers capital requirements for the SME portfolio.

The reason why, in the case of SMEs, there is a single intra-unit correlation for the modeled portfolio is that default rates were only available on an aggregated level.

Average correlation was obtained by weighting correlations by exposure for each sector.


---

**Figure 26: Intra-unit correlations by industry sector**

<table>
<thead>
<tr>
<th>Sector</th>
<th>SME intra-unit correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SFS</td>
</tr>
<tr>
<td>Retail</td>
<td>6.63%</td>
</tr>
<tr>
<td>Real estate development</td>
<td>12.20%</td>
</tr>
<tr>
<td>Construction</td>
<td>12.20%</td>
</tr>
<tr>
<td>Energy</td>
<td>5.91%</td>
</tr>
<tr>
<td>Manufacturing and textile</td>
<td>6.59%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>8.43%</td>
</tr>
<tr>
<td>Industrial production</td>
<td>13.17%</td>
</tr>
<tr>
<td>Tourism and services</td>
<td>7.17%</td>
</tr>
<tr>
<td>Average**</td>
<td>10.35%</td>
</tr>
</tbody>
</table>

Analysis and conclusions

Following are the capital charges obtained:

![Figure 27: Capital consumption by exposure for each industry sector](image)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>Monte Carlo 99.9%</th>
<th>Monte Carlo 99.97%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SFS</td>
<td>BIS II</td>
</tr>
<tr>
<td>Retail</td>
<td>3.80%</td>
<td>3.87%</td>
</tr>
<tr>
<td>Real estate development</td>
<td>6.54%</td>
<td>6.76%</td>
</tr>
<tr>
<td>Construction</td>
<td>5.60%</td>
<td>5.79%</td>
</tr>
<tr>
<td>Energy</td>
<td>3.08%</td>
<td>3.18%</td>
</tr>
<tr>
<td>Manufacturing and textile</td>
<td>4.81%</td>
<td>4.92%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>4.61%</td>
<td>4.75%</td>
</tr>
<tr>
<td>Industrial production</td>
<td>3.55%</td>
<td>3.64%</td>
</tr>
<tr>
<td>Tourism and services</td>
<td>5.61%</td>
<td>5.68%</td>
</tr>
</tbody>
</table>

It can be observed that for the real estate development, construction, tourism and other business sectors, the capital charges obtained using Monte Carlo on both regulatory and Spanish Financial System correlations are similar. This means that using one correlation or another does not have a significant impact.

In any case, these estimates are lower than those obtained using internal correlations. This variability in outcome reveals the sensitivity and importance of performing internal asset correlation estimates as part of the capital self-evaluation process.

![Figure 28: Impact on capital consumption at the 99.9% confidence level by industry sector and intra-unit correlation used](image)

![Figure 29: Impact on capital consumption at the 99.97% confidence level by industry sector and intra-unit correlation used](image)

Figure 30 shows estimates of capital according to the various models for the entire SME portfolio and with the different correlation estimates.

These estimates show that regulatory capital requirements are more conservative than those calculated using correlations.

![Figure 30: Capital consumption by exposure for different intra-unit correlations by portfolio](image)
obtained from Spanish Financial System information, but lower than those estimated based on internally calculated correlations. This emphasizes the importance of estimating correlations internally.

**Geographic concentration**

**Overview and objectives**

In light of the Geographic distribution of entities within the Spanish Financial System, it is appropriate to analyze the impact that geographic concentration can have on capital. This analysis has been conducted for the Mortgage portfolio since it is considered that this portfolio is more local in nature than the SME portfolio, since many SMEs (especially those above a certain size) undertake their business activities in different regions.

Thus, two portfolios which are identical in terms of probability of default, exposure and severity, can behave differently depending on the geographical region to which they belong, as their counterparties can have different correlation levels.

An intra-unit correlation for each geographical region was calculated using unemployment rate series (1991-2009)\(^54\) as a proxy to default rates. Each of these correlations was used in the model in order to calculate capital charges, assuming that the entire Mortgage portfolio is concentrated in the geographical region associated with each correlation.

That is, capital calculations are independent from each other: a separate calculation is performed by region, each under the assumption that the entire Mortgage portfolio pertains to such region. In this respect, the only value that changes from one calculation to another is the asset correlation value associated with the region for which the calculation is performed.

Regulatory capital and capital calculated using the economic capital model with the Spanish Financial System correlation, which is 14.22% (very similar to the regulatory 15%) are also estimated at the aggregated level. This last analysis is equivalent to having a portfolio distributed across the entire financial system, and thus across the entire country’s geography.

**Analysis and conclusions**

Using the intra-unit correlations pertaining to each geographical region in the model produces differences greater than 50% in capital consumption when applied at the 99.9% confidence level. The resulting variability in capital charges is caused by the fact that the different geographic regions have different levels of sensitivity to the cycle, which results in different estimates of correlation.

Therefore, these differences reveal the need to analyze Geographic concentration as part of the capital self-evaluation process.

**Concentration by risk segment**

**Overview and objectives**

All portfolios carry an inherent risk, and although when risk portfolios are added together the expected loss is the sum of all expected losses from all the risk portfolios, this is not usually the case with capital consumption. This can be clearly seen from an example depicting two portfolios. If much dependency exists between both portfolios, large (small) losses from the first portfolio will result in large (small) losses for the second

\(^54\)Advanced IRB with maturity adjustment and (in brackets) without maturity adjustment.

\(^5\)Source: INE. It should be noted that no public data on default rates by geographical region has been found.
portfolio. However, where there is little dependency (much diversification), large losses are intuitively expected to be compensated by moderate losses from the other portfolio, resulting in lower loss variability and therefore lower capital consumption.

The purpose of this test is to measure the impact of diversification between the two portfolios observed under the aggregation scheme described in the methodology section.

Aggregate capital resulting from applying the regulatory aggregation approach (i.e. direct addition) and the approach proposed is shown. For the latter, both the use of empirical correlation and a variation on this correlation by +15% and -15% are considered.

**Figure 31: Correlation between Mortgage and SME portfolios (losses)**

<table>
<thead>
<tr>
<th>Correlation between portfolios</th>
<th>51.38%</th>
<th>66.38%</th>
<th>81.38%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15% Empirical +15%</td>
<td>51.38%</td>
<td>66.38%</td>
<td>81.38%</td>
</tr>
</tbody>
</table>

**Analysis and conclusions**

Following are the capital consumption values obtained:

**Figure 32: Capital consumption by exposure based on correlation between portfolios**

<table>
<thead>
<tr>
<th></th>
<th>Basel II</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Advanced IRB</td>
</tr>
<tr>
<td>Direct addition of capital (Mortgages and SMEs)</td>
<td>3.88%</td>
<td>4.70%</td>
</tr>
<tr>
<td>Aggregation with correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>5.26%</td>
<td>3.11%</td>
</tr>
<tr>
<td></td>
<td>(3.82%)</td>
<td>3.70%</td>
</tr>
<tr>
<td>51.38%</td>
<td>3.52%</td>
<td>4.18%</td>
</tr>
<tr>
<td>66.38%</td>
<td>3.67%</td>
<td>4.36%</td>
</tr>
<tr>
<td>81.38%</td>
<td>3.81%</td>
<td>4.54%</td>
</tr>
</tbody>
</table>

It can be seen that, when diversification between portfolios is considered, the sum of the capital associated with both portfolios decreases by 5% from 3.88% to 3.67% under the economic capital model. It can also be observed that, if losses from both portfolios were totally independent, capital consumption would decrease by 20%, from 3.88% to 3.11%. In contrast, the regulatory model is not sensitive to this diversification between portfolios.

The effect of diversification by risk segment not only depends on the correlation between segments, but also on the number of portfolios and on their representativeness. Therefore, the impact of diversification will vary across entities depending on the distribution of their credit portfolios.

**General scenario**

**Overview and objectives**

The tests described above were used to analyze the individual impact on capital of each of the effects under consideration. In order to obtain an approximation to the overall impact of these effects, a test was conducted whereby stand-alone and diversified capital was calculated both separately and jointly for Mortgages and SMEs, resulting in the following joint impacts:

- Through-the-cycle adjustment of parameters in the initial sample.
- Procyclicality, assuming that through-the-cycle PD values are 10% higher than actual PD averages after removing procyclicality and applying the methodology that mitigates the impact of procyclicality on estimates, which, based on the above, amounts to reducing PD by 10%.
- Stochastic LGD (volatility and PD/LGD correlation) under the core scenario for the analyses performed in this connection.
- Rating migration (for SMEs) according to the analyses performed.
- Concentration:
  - Intra-unit correlation by portfolio based on regulatory criteria.
  - Inter-unit correlation between portfolios based on empirical analysis.

\[ \text{Estimated on the basis of the portfolios constructed.} \]

\[ \text{Which is consistent with a 95% confidence interval for the estimated parameter distribution.} \]
Analysis and conclusions

The following are the capital consumption values obtained:

Figure 33: Capital consumption in the general test

<table>
<thead>
<tr>
<th></th>
<th>Basel II</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Advanced IRB</td>
</tr>
<tr>
<td>Mortgages</td>
<td>2.93%</td>
<td>2.03%</td>
</tr>
<tr>
<td>SMEs</td>
<td>7.42%</td>
<td>6.75% (5.49%)</td>
</tr>
<tr>
<td>Aggregate</td>
<td>5.26%</td>
<td>4.48% (3.82%)</td>
</tr>
<tr>
<td>Mortgages - diversified</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>SMEs - diversified</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Applying the economic capital model to the aggregate portfolio results in a capital charge that is 13% higher than the regulatory requirement, as the impact of diversification and procyclicality does not suffice to offset the stochastic LGD and name concentration in SMEs.

With regard to the Mortgage portfolio, the procyclicality adjustment and diversification effect mitigate the increase in capital caused by stochastic LGD, resulting in capital charges under this model which are similar to regulatory estimates.

In contrast, capital consumption under this model for the SME portfolio appears to be higher than regulatory values, which set capital levels at 7.89% and 6.75% respectively, i.e. capital under this model is 17% higher than regulatory requirements. As can be observed from the above tests, this is mainly due to the effect of PD/LGD correlation and portfolio concentration.
Management Solutions is an international consultancy firm focusing on providing business, risk, financial, organizational and process-related advice, both in respect of functional components and in the implementation of related technologies.

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